Determination of resultant force and moment of light radiation pressure upon a perspective space telescope Millimetron

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The paper discusses problems related to the determination of the resultant force and moment of light radiation pressure force on the space structure of complex geometric shapes. The perspective space observatory "Millimetron" developed in the international partnership was taken as an example. This spacecraft will operate in a halo orbit around the Lagrange point L_2. For such structures, which are far from gravitating bodies, the main factor affecting the movement around the center of mass is a light pressure. The resultant moment of light pressure reduces observation time of remote space objects due to the need of unloading of reaction wheels by orientation engines. The basis of the proposed method is a tensor approach developed in several works. It was suggested to use a higher rank tensor approximation for resultant vector and moment than it was in works, in the form of series of tensors of increasing rank. The proposed approach has allowed writing down the explicit expressions for the resultant force and moment of light radiation pressure force on an idealized model of the spacecraft "Millimetron" ("Spektr-M"). This explicit calculation method for force and moment was compared with Monte-Carlo raytracing. It was shown that the accuracy of the description of the resultant force and moment of light pressure for proposed method and for Monte-Carlo simulation are the same but it is significantly less consuming in terms of computing resources, making this algorithm suitable for ballistic optimization and for using in onboard spacecraft control system.

Key Words: Solar Sail, Resultant Force, Resultant Moment, Light Pressure, Millimetron

Nomenclature

$\hat{\boldsymbol{n}}$	:	normal unit vector to the surface
$\hat{\boldsymbol{s}}$	:	light orientation vector directed from Sun
dA	:	infinitesimal surface area element
$\boldsymbol{r}$	:	position vector of surface element
B_m	:	Chebyshev coefficient of order of m
$\lfloor x \rfloor$	:	floor function of x
ε	:	emissivity of surface element
B	:	Lambertian coefficient, 2/3 for diffuse surface, for surface element
σ	:	Stephan-Boltzmann constant
T	:	temperature of surface element
ρ	:	reflectivity of surface element
s	:	coefficient of specularity for surface element
A	:	surface area
$d\boldsymbol{F}$	:	light pressure force on infinitesimal element of area
$d\boldsymbol{M}$	:	light pressure moment on infinitesimal element of area
$\boldsymbol{F}$	:	light pressure resultant force
$\boldsymbol{M}$	:	light pressure resultant moment
$P(R)$	:	light radiation pressure on ideally absorbing plate perpendicular to the light ray on the distance R from Sun
$N_{\max}$	:	number of terms in the series
J^n	:	force tensor of rank n for area element
L^n	:	moment tensor of rank n for area element
I^n	:	force tensor of rank n for the whole surface
K^n	:	moment tensor of rank n for the whole surface
J_A^n	:	first shape tensor for force
J_B^n	:	second shape tensor for force
L_A^n	:	first shape tensor for a moment
L_B^n	:	second shape tensor for a moment
E^2	:	unity tensor of the second rank
R^2	:	tensor representation of cross product
$[\boldsymbol{a}, \boldsymbol{b}]$	:	the dyadic product of vectors a and b
D^2	:	tensor representation of bidirectional reflection distribution function

1. Introduction

For the space vehicles in the deep space, the primary factor which is affecting their attitude is a moment from solar radiation pressure.[1)] It is important to develop the determination algorithm for light pressure moment. One of the well-developed methods is a collection of raytracing method (Monte-Carlo raytracing).[2-4)] However, these methods suffer from heavy consumption of computational resources so for the space satellite's onboard software we have to utilize another method for determination or approximation of light radiation pressure force and moment. There is a prospective tensor approach for determination of light radiation pressure developed by D.J. Scheeres and others,[5-10] however the tensor components should be recalculated if the structure changes the list of illuminated and shadowed elements. This method cannot be used for structures with self-shadowing and self-reflections. This study, as well as other works by another author,[11,12] utilizes the tensor approach with modifications to increase the quality of tensor approximation of light radiation pressure force and moment.

2. Mathematical model of light radiation pressure

2.1. Infinitesimal light pressure force

Let us consider the following visibility function which equals zero if the infinitesimal area of the surface is not illuminated:

$$V(\hat{\boldsymbol{n}},\hat{\boldsymbol{s}}) = \frac{\hat{\boldsymbol{n}}\cdot\hat{\boldsymbol{s}} - |\hat{\boldsymbol{n}}\cdot\hat{\boldsymbol{s}}|}{2}.$$

The absolute value function can be represented as a series of Chebyshev polynomials of the first kind:

$$|\hat{\boldsymbol{n}}\cdot\hat{\boldsymbol{s}}| = \frac{2}{\pi} - \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n T_{2n}(\hat{\boldsymbol{n}}\cdot\hat{\boldsymbol{s}})}{-1+4n^2} \approx \sum_{m=0}^{N_{max}} B_m(\hat{\boldsymbol{n}}\cdot\hat{\boldsymbol{s}})^{2m},$$

where:

$$B_m = -\frac{(-1)^m 4^{m+1}}{\pi(2m)!}\sum_{n=m}^{\left\lfloor\frac{N_{max}-1}{2}\right\rfloor}\frac{n(n+m-1)!}{(-1+4n^2)(n-m)!}. \quad (1)$$

It can be shown that the series for B_m diverges however for any finite N_{max} the series for absolute value converges on interval [-1,1].

By substitution of visibility function into the known equation for the infinitesimal force of light radiation pressure,[5] assuming only light force from emission only from illuminated surface, we can write the following relation:

$$d\boldsymbol{F} = \frac{P(R)}{2}(-2a_0\hat{\boldsymbol{n}} - a_1(\hat{\boldsymbol{n}}\cdot\hat{\boldsymbol{s}})\hat{\boldsymbol{s}} + a_2(\hat{\boldsymbol{n}}\cdot\hat{\boldsymbol{s}} - |\hat{\boldsymbol{n}}\cdot\hat{\boldsymbol{s}}|)\hat{\boldsymbol{n}} - a_3(\hat{\boldsymbol{n}}\cdot\hat{\boldsymbol{s}} - |\hat{\boldsymbol{n}}\cdot\hat{\boldsymbol{s}}|)^2\hat{\boldsymbol{n}})dA, \quad (2)$$

where a_0, a_1, a_2, a_3 – optical parameters:

$$\mathrm{a}_0 = \frac{\varepsilon B\sigma T^4}{P(R)};$$

$$\mathrm{a}_1 = 1-\rho s;$$

$$\mathrm{a}_2 = B\rho(1-s);$$

$$\mathrm{a}_3 = \rho s.$$

2.2. Light pressure force

2. By substitution of Eq. (1) into Eq. (2), following the procedure of L. Rios-Reyes and D.J. Scheeres, and finally, by integrating over all surface, it is possible to create the tensor series for light radiation pressure resultant force and moment:

$$\boldsymbol{F} = P(R)\left(I^0\hat{\boldsymbol{s}} + I^1 + \sum_{n=2}^{N_{max}} I^n\cdot\underbrace{\hat{\boldsymbol{s}}\cdot\ldots\cdot\hat{\boldsymbol{s}}}_{n-1}\right). \quad (3)$$

$$\boldsymbol{M} = P(R)\left(K^{2,0}\cdot\hat{\boldsymbol{s}} + K^1 + \sum_{n=2}^{N_{max}} K^n\cdot\underbrace{\hat{\boldsymbol{s}}\cdot\ldots\cdot\hat{\boldsymbol{s}}}_{n-1}\right). \quad (4)$$

Where:

$$I^n = \int_A J^n dA;$$

$$K^n = \int_A L^n dA;$$

$$J^0 = \frac{a_1}{2}B_0;$$

$$J^1 = -(a_0 + a_2B_0)\hat{\boldsymbol{n}};$$

$$J^2 = \left(\frac{1}{2}a_2 + a_3B_0\right)J_A^2;$$

$$J^3 = \frac{1}{2}(-a_1J_B^3 - 2a_3J_A^3 - B_1a_2J_A^3);$$

$$J^n = \frac{1}{2}\left(-B_{\frac{n-1}{2}}\frac{1-(-1)^n}{2}a_2J_A^n + B_{\frac{n-2}{2}}\frac{1+(-1)^n}{2}(a_1J_B^n + 2a_3J_A^n)\right), \quad n>3;$$

$$J_A^n = \underbrace{[\hat{\boldsymbol{n}},\ldots,\hat{\boldsymbol{n}}]}_{n};$$

$$J_B^n = \left[\underbrace{[\hat{\boldsymbol{n}},\ldots,\hat{\boldsymbol{n}}]}_{n-2}, E^2\right];$$

$$L^{2,0} = \frac{a_1}{2}B_0R^2;$$

$$L^1 = -(a_0 + a_2B_0)(R^2\cdot\hat{\boldsymbol{n}});$$

$$L^2 = \left(\frac{1}{2}a_2 + a_3B_0\right)L_A^2;$$

$$L^3 = \frac{1}{2}(-a_1L_B^3 - 2a_3L_A^3 - B_1a_2L_A^3);$$

$$L^n = \frac{1}{2}\left(-B_{\frac{n-1}{2}}\frac{1-(-1)^n}{2}a_2L_A^n + B_{\frac{n-2}{2}}\frac{1+(-1)^n}{2}(a_1L_B^n + 2a_3L_A^n)\right), \quad n>3;$$

$$L_A^n = \left[\underbrace{[\hat{\boldsymbol{n}},\ldots,\hat{\boldsymbol{n}}]}_{n-1}, (R^2\cdot\hat{\boldsymbol{n}})\right];$$

$$L_B^n = \left[\underbrace{[\hat{\boldsymbol{n}},\ldots,\hat{\boldsymbol{n}}]}_{n-2}, R^2\right],$$

where R^2 is a second-rank tensor representation of a cross vector product, e.g. for any vector $\boldsymbol{a}$: $\boldsymbol{r}\times\boldsymbol{a} = R^2\cdot\boldsymbol{a}$. The Eq. (3) and (4) represent the light pressure resultant force and moment for any optically convex space structure.[11] The tensor components can be calculated once since they are not depending of light a orientation vector. As it shown in 12), one can write the similar tensor series for force and moment for non-convex structures by introducing the approximation of $\hat{\boldsymbol{s}}$ using BRDF tensor of a second rank D^2:

$$\boldsymbol{s} = D^2 \cdot \hat{\boldsymbol{s}}.$$

The resulting approximation can be obtained by two ways: by exact calculation of tensor components over whole structure and by approximation of these components using known values of force and moment which can be calculated be another method. In this paper, we used the Monte-Carlo raytracing method to calculate the known values of light radiation pressure.

5. Resultant force and moment on Millimetron Space Telescope

As an example, we calculated the resultant force and moment of light radiation pressure upon a perspective space observatory Millimetron (fig. 1).

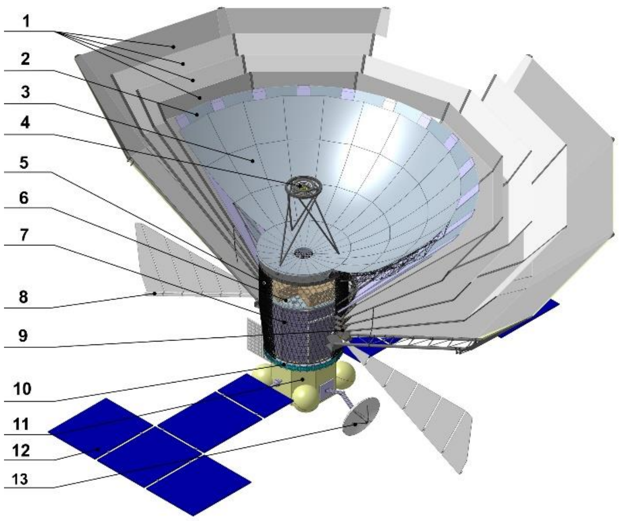

Fig. 1. Millimetron space observatory concept. 1 - Sun shields; 2 - Cryo-shield; 3 – Primary mirror's petal; 4 - Secondary mirror; 5 - Central part of Primary mirror; 6 - Cryo-container; 7 - Heat exchanger (radiator); 8 - Warm container; 9 - Sunshields supporting truss; 10 - Adapter ring; 11 - Service module; 12 - Solar power array; 13 - High gain antenna.

Firstly, we calculated the resultant force and moment in 60 different orientation cases using Monte-Carlo raytracing. The moment origin was placed in the center of the external heat shield. The number or rays were equal to 1000000 in all cases. The whole surface was considered specular.

Secondly, we calculated the approximated components of tensors for force and moment using the least squares method as it shown in fig. 2 and 3. The approximation rank N_{max} was equal to 6.

7. Conclusion

The proposed method can be used for approximation of light radiation pressure force and moment for space structures with complex geometric shapes. It is still important to regularize the divergent series Eq. (1). This method can be used in the satellite's onboard software: the tensor coefficients can be calculated on the ground or updated according to after the measurements of angular movements. After this, the onboard software can recalculate the light pressure force and moment for the purposes of measurement filtration and for the angular stabilization.

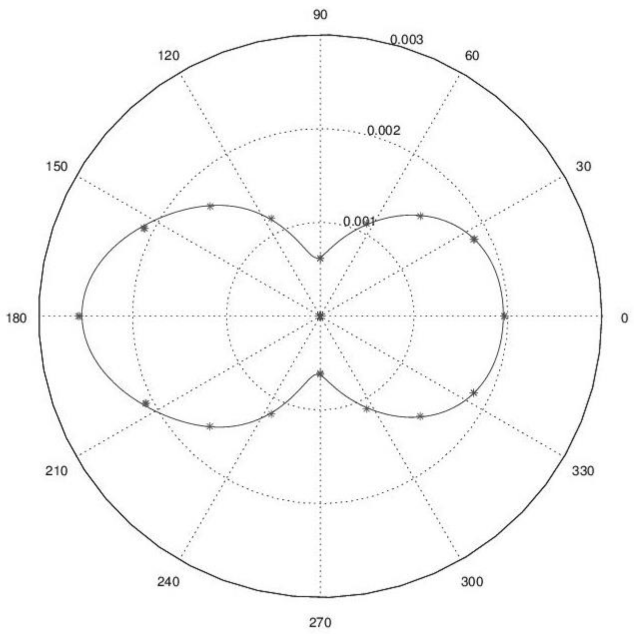

Fig. 2. Approximation of resultant force of light radiation pressure upon Millimetron depending on rotation angle in the radiator's plane (solid line) and results of Monte-Carlo simulation (dots), Newtons.

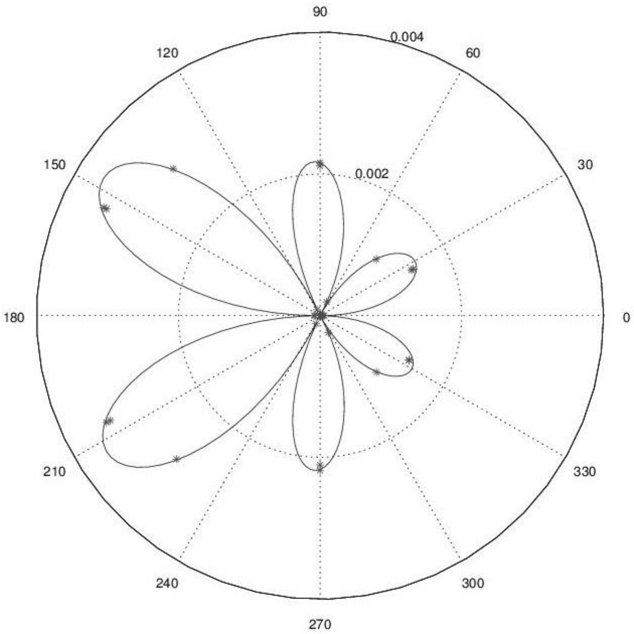

Fig. 3. Approximation of resultant moment of light radiation pressure upon Millimetron depending on rotation angle in the radiator's plane (solid line) and results of Monte-Carlo simulation (dots), Newtons x meters.

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